Computational morphology.
Day 1. Theory of formal languages.

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Outline of the course

- Day 1: What is computational morphology? Theory of formal languages: regular expressions and finite automata.
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Day 2: Finite transducers. Their application to natural languages.
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- Day 4: Applying hidden Markov models to morphological analysis.
Computational morphology. Day 1. Theory of formal languages.

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- Day 2: Finite transducers. Their application to natural languages.
- Day 4: Applying hidden Markov models to morphological analysis.
- Day 5: Other methods and models for morphological analysis.
Day 1 outline

- What is computational morphology?
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- Regular expressions.
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- Finite automata.
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- What is computational morphology?
- Regular expressions.
- Finite automata.
- Finite automata for linguistic phenomena.
What is morphology?

“Morphology is the study of the forms of words, and the ways in which words are related to other words of the same language.” (R. Andersen).

“Morphology is the part of linguistics which studies the word in all its relevant aspects.” (I. A. Melchuk).
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Informally, morphology studies:

- How the word changes in different contexts (word inflection).
- What factors determine these changes (morphological categories).
- What parts of the word reflect these changes (morpheme analysis).
Tasks of computational morphology

Basic tasks of computational morphology:

- Morphological analysis (tagging):
  
  *lirons* (“(we will) read”) $\rightarrow$ lire+Fut+Pl+1
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- Morphological synthesis:
  
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- Lemmatization:
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- Paradigm detection:
  
  \[ \text{parler} \mapsto \text{parl-er, parl-e, parl-es, parl-e, parl-ons, parl-ez, parl-ent} \]
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Context-dependent morphology

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- But lemmatization and analysis DO!
- *parent* → *parent+NOUN+Masc+Sg*:
  
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  \begin{align*}
  &Mon &parent &es &grand \\
  &"My &parent &is &tall"
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  *Les défenseurs parent tous les tirs*  
  “The defenders block all the shots”
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  \item \textit{Les défenseurs parent tous les tirs}  
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  \end{itemize}

- The effect of context is far more strong in highly inflective languages (Russian, Czech etc.).
Applications

- Machine translation:

  \[ \text{Pete bought a book} \rightarrow \text{Petya kupil knigu} \]

  \[
  \begin{align*}
  \text{bought} & \quad \downarrow \\
  \text{buy} + \text{Past} & \quad \downarrow \quad \text{(with single masculine object)} \\
  \text{kupit’} + \text{Past} + \text{Sg} + 3 + \text{Masc} & \quad \downarrow \\
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- **Language modelling:** making a probability model more sparse.
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  \downarrow \\
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- Information retrieval.
- Language modelling: making a probability model more sparse.
- Actually, morphological tagging is a preprocessing step for almost all NLP tasks.
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Now let us describe a word...
A word includes at least one vowel and arbitrary number of consonants.
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Now let us describe a word...
A word includes at least one vowel and arbitrary number of consonants.
Answer: $(C|V)^*V(C|V)^*$ where | stands for OR.
We wish to describe the syllable structure of the word more carefully.
More complex examples

- We wish to describe the syllable structure of the word more carefully.
- We add the condition that exactly one syllable is stressed $V_0$ and the syllables are separated by hyphens (−).
- Then a stressed syllable is $C^* V_0 C^*$. 
More complex examples

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- We add the condition that exactly one syllable is stressed $V_0$ and the syllables are separated by hyphens ($-\,$).
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- Let us separate two cases. **First case**: stressed syllable is the last one.
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  - All unstressed syllables are followed by a hyphen. That is $C^*VC^*-\ (V$ stands for unstressed).
  - We have an arbitrary number of such groups $(C^*VC^*-)^*$ followed by a stressed syllable $C^*V_0C^*$.
  - Concatenating, we obtain $(C^*VC^*-)^*C^*V_0C^*$.
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- Let us separate two cases. First case: stressed syllable is the last one. $(C^*V_0C^*−)^*C^*V_0C^*$
- Second case: stressed syllable is not the last one.
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- **Second case:** stressed syllable is not the last one.
  - Arbitrary number of hyphenated unstressed syllables, followed by a hyphenated stressed syllable,
  - followed by arbitrary number of hyphenated unstressed syllables, followed by an unstressed syllable.
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  - Arbitrary number of hyphenated unstressed syllables, followed by a hyphenated stressed syllable,
  - followed by arbitrary number of hyphenated unstressed syllables, followed by an unstressed syllable.
  - Together, $(C^*VC^*-)^*C^*V_0C^* - (C^*VC^*-)^*C^*VC^*$. 
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- The answer is $((C^* V C^* -) * C^* V_0 C^*) | ((C^* V C^* -) * C^* V_0 C^* - (C^* V C^* -) * C^* V C^*)$. 
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- The answer is $((C^*VC^*−)^*C^*V_0C^*)|((C^*VC^*−)^*C^*V_0C^* − (C^*VC^*−)^*C^*VC^*)$.
- Regrouping (? is “can be present or not”):
  $((C^*VC^*−)^*C^*V_0C^*)((−(C^*VC^*−)^*C^*VC^*)?)$. 
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- We add the condition that exactly one syllable is stressed \( V_0 \) and the syllables are separated by hyphens (−).
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- Let us separate two cases. **First case**: stressed syllable is the last one. \( (C^* V_0 C^*−) C^* V_0 C^* \)
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- The answer is \( ((C^* V C^*−) C^* V_0 C^*) | ((C^* V C^*−) C^* V_0 C^* − (C^* V C^*−) C^* V C^*) \).
- Regrouping (‘? is “can be present or not”’): \( ((C^* V C^*−) C^* V_0 C^*) | (−(C^* V C^*−) C^* V C^*) ?) \).
- Another variant: \( (C^* V C^*−) C^* V_0 C^* (−C^* V C^* )^* \).
Examples for morphology

- Spanish verb infinitive ends with -ar, -ir, -er which is followed by -se in case of reflexive verbs.
Examples for morphology

- Spanish verb infinitive ends with *-ar,-ir,-er* which is followed by *-se* in case of reflexive verbs.
- It is simple: \((C|V)^*(a|i|e)r(se)?\).
- \(C\) is an arbitrary consonant (just join all consonants with |) and \(V\) is a vowel.
Examples for morphology

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- -s cannot appear after e preceded by a consonant (sky → skies).
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- *-es* follows a sibilant (*s, x, z, ch, sh*).
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But `-s` must be avoided after `s, x, z, ch, sh, C_y`, where `C` is arbitrary consonant.

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- For this task it is easier to parse witches as witche+s, not to deal with -es.
- But -s must be avoided after s, x, z, ch, sh, Cy, where C is arbitrary consonant.
- But regular expression cannot express negative patterns.
- Solution: list all that is allowed.
A plural form is a stem followed by -s, where a stem can be anything that:

- Ends with vowel not equal to y: \((C | V)^* (a | e | i | o | u)\).
- Ends with vowel + y: \((C | V)^* Vy\).
- Contains a vowel and ends with a consonant not equal to s, x, z, h (let \(C'\) denote their complete list): \((C | V)^* V (C | V)^* C'\).
- Contains a vowel and ends with h or \(C''h\), where \(C''\) stands for all consonants except s, c: \((C | V)^* V (C | V)^* C''? h\).

Grouping all together: \((C | V)^* (((a | e | i | o | u) | V (C | V)^*)^* (C' | C''? h))^* s\).
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- Ends with vowel not equal to y: \((C|V)*(a|e|i|o|u)\).
- Ends with vowel+y: \((C|V)*V\).
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  - Contains a vowel and ends with h or \(C'' h\), where \(C''\) stands for all consonants except s, c: \((C|V)*V(C|V)*C''?h\)
  - Grouping all together: \((C|V)*((a|e|i|o|u|Vy)V(C|V)*(C'|C''?h))s\).
Formal definitions

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- Languages — sets of words: $L \subseteq \Sigma^*$.
- Operations on languages:
  - Boolean operations: $L_1 \cup L_2$, $L_1 \cap L_2$, $L_1 - L_2$, $\overline{L}$ (complement).
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  - **Power** $L^k = \underbrace{L \cdot \ldots \cdot L}$, $L^0 = \{\varepsilon\}$, $L^1 = L$. $k$ times
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  - Power $L^k = \underbrace{L \cdot \ldots \cdot L}_{k \text{ times}}$. $L^0 = \{ \varepsilon \}$, $L^1 = L$.
  - Iteration (Kleene star): $L^* = \bigcup_{k=0}^{\infty} L^k$. 

\[ \{a, b\}^* = \{ \varepsilon, a, b, aa, ab, ba, bb, \ldots \} \]
Computational morphology. Day 1. Theory of formal languages.

Theory of formal languages

Regular languages

Formal definitions

- Alphabet — arbitrary finite set $\Sigma$, its elements — letters.
- Words — finite sequences of letters, the set of words — $\Sigma^*$.
- $\varepsilon$ — empty word.
- $\cdot$ — concatenation of words, $ad \cdot bc = adbc$.
- Languages — sets of words: $L \subseteq \Sigma^*$.
- Operations on languages:
  - Boolean operations: $L_1 \cup L_2$, $L_1 \cap L_2$, $L_1 - L_2$, $\overline{L}$ (complement).
  - Concatenation: $L_1 \cdot L_2 = \{w_1 \cdot w_2 | w_1 \in L_1, w_2 \in L_2\}$.
  - Power $L^k = \underbrace{L \cdot \ldots \cdot L}_{k \text{ times}}$. $L^0 = \{\varepsilon\}$, $L^1 = L$.
  - Iteration (Kleene star): $L^* = \bigcup_{k=0}^{\infty} L^k$.
- $\{a, b\}^* = \{a, b\}^0 \cup \{a, b\}^1 \cup \{a, b\}^2 \cup \ldots = \{\varepsilon, a, b, aa, ab, ba, bb, \ldots\}$. 
Regular expressions: what is it formally

- We distinguish regular expression $\alpha$ and its language $L(\alpha)$.
- For example, if $\alpha = (a|b)(a|c)$, then $L(\alpha) = \{aa, ac, ba, bc\}$. 
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- Let some alphabet $\Sigma$ be fixed.
- Regular expressions ($\text{Reg}(\Sigma)$):
  - Any $a \in \Sigma$ is a regular expression, $L(a) = \{a\}$. 
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  - For all $\alpha, \beta \in \text{Reg}(\Sigma)$ also $(\alpha|\beta) \in \text{Reg}(\Sigma)$,
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Common conventions:

- $\alpha^+ = \alpha \alpha^*$ (positive iteration),
- $\alpha^? = (\alpha|1)$ (optional).
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Common conventions: $\alpha^+ = \alpha \alpha^*$ (positive iteration), $\alpha? = (\alpha|1)$ (optionality).
Regular expressions: what is it formally

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- Common conventions: $\alpha^+ = \alpha \alpha^*$ (positive iteration),
  $\alpha? = (\alpha|1)$ (optionality).
- Regular languages: languages that can be expressed by regular expressions.
Examples of regular languages

- Words with exactly two $a$-s (alphabet $a, b$): $(a|b)^* a(a|b)^* a(a|b)^*$. 
Examples of regular languages

- Words with exactly two $a$-s (alphabet $a$, $b$): $(a|b)^*a(a|b)^*a(a|b)^*$.
- Words with even number of $a$-s (alphabet $a$, $b$):
  $((a|b)^*a(a|b)^*a)^*(a|b)^*$.
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- Words with exactly two $a$-s (alphabet $a, b$): $(a|b)^* a(a|b)^* a(a|b)^*$.
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- Words with odd number of $a$-s (alphabet $a, b$): Exercise.
Examples of regular languages

- Words with exactly two a-s (alphabet a, b): \((a|b)^* a(a|b)* a(a|b)^*\).
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- Words with odd number of a-s (alphabet a, b): Exercise.
- \(a\) is immediately followed by \(b\) (alphabet a, b, c): \((b|c|ab)^*\).

- a is immediately preceded by b: Exercise.
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- After every \(a\) \(b\) occurs earlier than \(c\) (alphabet \(a, b, c, d\)):
  
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- After every $a$ $b$ occurs earlier than $c$ (alphabet $a$, $b$, $c$, $d$):
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- Left to $a$ $b$ occurs closer than $c$: Exercise.
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- Words with odd number of a-s (alphabet a, b): Exercise.
- a is immediately followed by b (alphabet a, b, c): \((b|c|ab)^*\).
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- After every a b occurs earlier than c (alphabet a, b, c, d):
  \((ad^* b|b|c|d)^*\).
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- No repeating letters (alphabet a, b):
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- No repeating letters (alphabet $a, b$): $b?(ab)^*a?$. 
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  $H = a(ba)^* b? | b(ab)^* a?$.
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\[((a|b)^* a(a|b)^* a)^* (a|b)^*\).

Words with odd number of a-s (alphabet a, b): Exercise.

a is immediately followed by b (alphabet a, b, c): \((b|c|ab)^*\).

a is immediately preceded by b: Exercise.

After every a b occurs earlier than c (alphabet a, b, c, d):
\((ad^* b|b|c|d)^*\).

Left to a b occurs closer than c: Exercise.

No repeating letters (alphabet a, b): \(b?(ab)^* a?\).

Non-empty word with no repetitions:
\(H = a(ba)^* b?|b(ab)^* a?\).

No repeating letters (alphabet a, b, c): \(H?(cH)^* c?\).
Exercise: vowel harmony

- Words that have at least one letter among $V, V_1, V_2$, but not $V_1$ and $V_2$ together.
Exercise: vowel harmony

- Words that have at least one letter among $V, V_1, V_2$, but not $V_1$ and $V_2$ together.
- Explanation: $V_1$ and $V_2$ are disharmonic types of vowels (say, soft and round). $V$ are neutral vowels, $C$ are consonants.
Exercise: vowel harmony

- Words that have at least one letter among \( V, V_1, V_2 \), but not \( V_1 \) and \( V_2 \) together.
- Explanation: \( V_1 \) and \( V_2 \) are disharmonic types of vowels (say, soft and round). \( V \) are neutral vowels, \( C \) are consonants.

\[
C^*(V|V_1)(C|V|V_1)^*|C^*(V|V_2)(C|V|V_2)^*.
\]
Exercise: vowel harmony

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$$C^* (V | V_1) (C | V | V_1)^* | C^* (V | V_2) (C | V | V_2)^*.$$ 

Exercise: Turkish infinitives.

In Turkish there are 8 vowels:

<table>
<thead>
<tr>
<th></th>
<th>Front</th>
<th>Back</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>e i</td>
<td>a i</td>
</tr>
<tr>
<td>Round</td>
<td>ü ö</td>
<td>u o</td>
</tr>
</tbody>
</table>

Infinitive is formed by suffix -mek/-mak attached to verb stem, where $e$ appears if the last vowel of stem is front and $a$ – if it is back. Write a regular expression for Turkish infinitives.
Finite automata

- Regular expressions are convenient to describe patterns.
Finite automata

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Example: $a(a|b|c)^*b(a|b|c)$. 
Finite automata

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- Example: $a(a|b|c)^*b(a|b|c)$.
- How we can process it:
  - Read the first letter, check that it is $a$, otherwise reject.
Regular expressions are convenient to describe patterns.
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Example: \( a(a|b|c)^* b(a|b|c) \).
How we can process it:
- Read the first letter, check that it is \( a \), otherwise reject.
- Read the letters until the penultimate letter appears.
Regular expressions are convenient to describe patterns.

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Example: $a(a|b|c)^*b(a|b|c)$.

How we can process it:

- Read the first letter, check that it is $a$, otherwise reject.
- Read the letters until the penultimate letter appears.
- Check that it is $b$.
- Check that exactly one letter remains.
Regular expressions are convinient to describe patterns.
But there is no way to check that a word satisfies to an expression.
Example: \(a(a \mid b \mid c)^* b(a \mid b \mid c)\).
How we can process it:
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- Read the letters until the penultimate letter appears.
- Check that it is \(b\).
- Check that exactly one letter remains.

Schematically:

That is finite automaton.
Finite automata

- Finite automaton consists of:
  - Final set of states $Q$.
  - Alphabet $\Sigma$.  
  - Initial state $q_0$.
  - Set of (possibly multiple) final states $F \subseteq Q$.

Every edge has its label. The label of a path is the concatenation of its edge labels.

Automaton $A$ accepts language $L(A)$ of all words that label paths from initial state to some final.
Finite automaton consists of:

- Final set of states $Q$.
- Alphabet $\Sigma$.
- Set of transitions (edges) $\Delta \subseteq Q \times \Sigma^* \times Q$:

$$ q_1 \xrightarrow{w} q_2 \quad \langle q_1, w \rangle \rightarrow q_2 $$

Initial state $q_0$.

Set of (possibly multiple) final states $F \subseteq Q$.

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  \[
  \begin{align*}
  q_1 & \xrightarrow{w} q_2 \\
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Finite automata: examples

- A syllable: states check vowel presence.

![Finite automaton diagram](image-url)
Finite automata: examples

- A syllable: states check vowel presence.

- Even number of a-s, alphabet a, b. States check parity of a-s.
Finite automata: examples

- A syllable: states check vowel presence.

```
\begin{array}{c}
\text{c} \\
\rightarrow q_0 \\
\text{V} \\
\rightarrow q_1 \\
\text{c, V}
\end{array}
```

- Even number of a-s, alphabet a, b. States check parity of a-s.

```
\begin{array}{c}
b \\
\rightarrow a \\
a \\
a
\end{array}
```

- Every a is immediately followed by b, alphabet a, b, c.

```
\begin{array}{c}
b, c \\
\rightarrow a \\
b \\
b
\end{array}
```
Finite automata: examples

- Every $a$ is immediately preceded by $b$, alphabet $a, b, c$. 

$$
\begin{array}{c}
\text{ Every } a \text{ is immediately preceded by } b, \text{ alphabet } a, b, c. \\
\end{array}
$$
Finite automata: examples

- Every $a$ is immediately preceded by $b$, alphabet $a, b, c$.

- To the right of every $a$ occurs $b$ with no $a, c$ between them, alphabet $a, b, c, d$. 
Finite automata: examples

No repeating letters, alphabet $a, b, c$. States correspond to letters:
Finite automata: examples

- Word syllabification: each syllable contains exactly one vowel and exactly one vowel is stressed, syllables are separated by hyphens.
Finite automata: examples

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- States check two conditions:
  - There was a vowel in current syllable (the first coordinate).
**Finite automata: examples**

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Word syllabification: each syllable contains exactly one vowel and exactly one vowel is stressed, syllables are separated by hyphens.

States check two conditions:
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- There was a stressed vowel (the second coordinate).
Finite automata: English plural

- All plural forms can be decomposed as stem + s, where

  - A stem is anything with at least one vowel, but not ending with:
    - -s, -x, -z, -sh, -ch, -zh (sibilants).

- Automaton for all possible stems ($C_0 = C - \{s, x, z, c, h\}$, $C_1 = C_0 \cup \{s, x, z\}$):
Finite automata: English plural

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Automaton for all possible stems

\( C_0 = C - \{s, x, z, c, h\} \), \( C_1 = C_0 \cup \{s, x, z\} \):
Theorem

*Every automata language is recognized by an automaton with single letter labels.*
Theorem

Every automata language is recognized by an automaton with single letter labels.

Sketch of the proof

- Split all labels of length ≥ 2 by inserting additional states.
- Now we have only letters and ε as labels.
Properties of finite automata

Theorem

*Every automata language is recognized by an automaton with single letter labels.*

Sketch of the proof

- Split all labels of length $\geq 2$ by inserting additional states.
- Now we have only letters and $\varepsilon$ as labels.
- Add an edge $\langle q_1, a \rangle \rightarrow q_2$ if there exist states $q_3, q_4$ such that $(\langle q_3, a \rangle \rightarrow q_4) \in \Delta$ and there are $\varepsilon$-paths from $q_1$ to $q_3$ and from $q_4$ to $q_2$. 

Mark as terminal all states from which terminal states are $\varepsilon$-reachable.

Now remove all $\varepsilon$-paths.
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An automaton with one-letter labels is deterministic if no state has two outcoming edges with the same label.

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Every automata language can be recognized by deterministic automata.
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- New automaton states are sets of old states.
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- New automaton states are sets of old states.
- An edge labeled by $a$ leads from set $Q_1$ to $Q_2$ if $Q_2$ contains exactly the states reachable from $Q_1$ by $a$. 
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- Final states: subsets containing at least one old final state.
Kleene theorem

Theorem

*The classes of automata and regular languages are the same.*
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Sketch of the proof

- We should transform every finite automaton to regular expression and every regular expression to finite automaton.
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- Regular languages are constructed from primitives by means of concatenation, union and iteration.
- Primitive regular languages (singletons and empty language) are certainly automata.
- We should prove that regular operations preserve automata languages.
Kleene theorem

**Theorem**

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**Sketch of the proof**

Concatenation: \( L_1 = L(M_1), L_2 = L(M_2) \rightarrow L_1 \cdot L_2 = L(M) \)
The classes of automata and regular languages are the same.

Sketch of the proof:
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Sketch of the proof

Union: $L_1 = L(M_1), L_2 = L(M_2) \rightarrow L_1 \cup L_2 = L(M)$
Theorem

The classes of automata and regular languages are the same.

Sketch of the proof

Iteration: \( L_1 = L(M_1) \), \( L_1^* = L(M) \)

\[
M_1 \quad \Rightarrow \quad M_1
\]
Properties of automata languages

Theorem

*The class of automata languages is closed under complement.*

Sketch of the proof

Consider the deterministic automaton for language \( L \).

Complete it: add a new sink state \( q' \).

If a state \( q_1 \) does not have an outgoing edge labeled by letter \( a \),

add an edge \( \langle q_1, a \rangle \rightarrow q' \).

Add edge \( \langle q', a \rangle \rightarrow a \) for every letter \( a \).

Now for every \( q_1 \in Q \), \( a \in \Sigma \) there is an edge of the form \( \langle q_1, a \rangle \rightarrow q_2 \).

Consequently, every word \( w \) leads from \( q_0 \) to exactly one state:

- terminal if \( w \in L \)
- non-terminal if \( w \notin L \).

Switching non-terminal and terminal states yields automaton for the complement.
Properties of automata languages

Theorem

The class of automata languages is closed under complement.

Sketch of the proof

- Consider the deterministic automaton for language $L$. 

Complete it: add a new sink state $q'$. If a state $q_1$ does not have an outgoing edge labeled by letter $a$, add an edge $\langle q_1, a \rangle \rightarrow q'$. Add edge $\langle q', a \rangle \rightarrow a$ for every letter $a$. Now for every $q_1 \in Q$, $a \in \Sigma$ there is an edge of the form $\langle q_1, a \rangle \rightarrow q_2$. Consequently, every word $w$ leads from $q_0$ to exactly one state: terminal if $w \in L$ and non-terminal if $w \in \overline{L}$. Switching non-terminal and terminal states yields automaton for the complement.
Properties of automata languages

**Theorem**

*The class of automata languages is closed under complement.*

**Sketch of the proof**

- Consider the deterministic automaton for language $L$.
- Complete it: add a new sink state $q'$.
- If a state $q_1$ does not have an outgoing edge labeled by letter $a$, add an edge $\langle q_1, a \rangle \to q'$.
### Properties of automata languages

#### Theorem

*The class of automata languages is closed under complement.*

#### Sketch of the proof

1. Consider the deterministic automaton for language $L$.  
2. Complete it: add a new sink state $q'$.  
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Properties of automata languages

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- Consequently, every word $w$ leads from $q_0$ to exactly one state: terminal if $w \in L$ and non-terminal if $w \in \overline{L}$.
- Switching non-terminal and terminal states yields automaton for the complement.
Properties of automata languages

**Theorem**

*The class of automata languages is closed under intersection.*

**Sketch of the proof**
Properties of automata languages

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The class of automata languages is closed under intersection.

Sketch of the proof

- Easy variant: \( L_1 \cap L_2 = \overline{L_1} \cup \overline{L_2} \).
Properties of automata languages

**Theorem**

The class of automata languages is closed under intersection.

**Sketch of the proof**

- **Easy variant:** $L_1 \cap L_2 = \overline{L_1 \cup L_2}$.
- **Complex (but effective) variant:** consider complete deterministic automata $M_1$ for $L_1$ and $M_2$ for $L_2$.
- Let $Q_1, Q_2$ be their sets of states, $q_{01}, q_{02}$ be initial states and $F_1, F_2$ be sets of final states.
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  - Let \( Q_1, Q_2 \) be their sets of states, \( q_{01}, q_{02} \) be initial states and \( F_1, F_2 \) be sets of final states.
  - Consider a new automaton whose states are pairs \( \langle q_1, q_2 \rangle \), \( q_1 \in Q_1, q_2 \in Q_2 \).
**The class of automata languages is closed under intersection.**

**Sketch of the proof**

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- Its start state is \( \langle q_{01}, q_{02} \rangle \).
- On the first coordinate it operates like \( M_1 \), on the second like \( M_2 \).
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- Its start state is \( \langle q_{01}, q_{02} \rangle \).
- On the first coordinate it operates like \( M_1 \), on the second like \( M_2 \).
- Finite states are pairs of final states (the automaton accepts iff it accepts for both coordinates).
Finite automata are closed under a couple of operations.
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Moreover, this closure is effective: corresponding automata are built algorithmically.
Recursive construction of automata

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- Therefore we may combine automata just as regular expressions, but with more operations.
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Therefore we may combine automata just as regular expressions, but with more operations.
For example, the automata for English plural can be expressed as:

$$\left( L_{sib} \cdot es \right) \cup \left( \left( \overline{L_{sib}} \cap L_C \right) \cup L_{Cy} \cup L_V \right) \cdot s,$$

where

- $L_{sib}$ — words ending with sibilant.
- $L_C$ — words ending with consonant.
- $L_{Cy}$ — words ending with consonant+y.
- $L_V$ — words ending with vowel (not y).
Finite automata are closed under a couple of operations. Moreover, this closure is effective: corresponding automata are built algorithmically. Therefore we may combine automata just as regular expressions, but with more operations. For example, the automata for English plural can be expressed as:

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The basic languages are the automata ones; the automaton for the whole expression could be constructed recursively.
### Turkish infinitive

**Construct a finite automaton for Turkish infinitive**

- Infinitive has the form stem + $mE_k$.
- Placeholder $E$ is filled by $e$ if the stem ends with $e$, $i$, $ö$, $ü$ and $a$ if it ends with $a$, $i$, $o$, $u$. 

Turkish infinitive

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- \(M_1\) is the automaton for expression \(C^*V(C|V)^*m(a|e)k\) (it is easy to construct it).
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- \(M_1\) is the automaton for expression \(C^* V (C|V)^* m(a|e)k\) (it is easy to construct it).
- \(M_2\) checks the condition for vowels:

\[ C, V \xrightarrow{e, i, ö, ü} C, V \xrightarrow{e} \]
\[ C, V \xrightarrow{a, i, o, u} C, V \xrightarrow{a} \]

- \(M_1 \cap M_2\) is the required automaton.
Recursive construction of automata

Turkish infinitive

Construct a finite automaton for Turkish passive infinitive

- Infinitive has the form stem + X + mEk.
- Placeholder E is filled by e if the stem ends with e, i, ö, ü and a if it ends with a, i, o, u.
- Suffix X is -n if the stem ends with vowel, -In if the stem ends with l and -Il otherwise.
- Placeholder I equals i after a, i; u after u, o; i after e, i; ü after ü, ö.
Where to get presentations


For the next day:
Install (simply download and unpack) finite-state compiler FOMA from https://code.google.com/archive/p/foma/.