Computational morphology. Day 1. Theory of formal languages.

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- Day 5: Other methods and models for morphological analysis.

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- Finite automata.
- Finite automata for linguistic phenomena.

What is morphology?

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Informally, morphology studies:

- How the word changes in different contexts (word inflection).
- What factors determine these changes (morphological categories).
- What parts of the word reflect these changes (morpheme analysis).

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- The effect of context is far more strong in highly inflective languages (Russian, Czech etc.).

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- Actually, morphological tagging is a preprocessing step for almost all NLP tasks.

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- Now let us describe a word...
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- Answer: $(C|V)^*V(C|V)^*$ where $|$ stands for OR.

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  - We have an arbitrary number of such groups $(C^* VC^* -)^*$ followed by a stressed syllable $C^* V_0 C^*$.
  - Concatenating, we obtain $(C^* VC^* -)^* C^* V_0 C^*$.

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  - Together, $(C^* VC^* -)^* C^* V_0 C^* - (C^* VC^* -)^* C^* VC^*$.

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- Another variant: $(C^*VC^*-)^*C^*V_0C^*(-C^*VC^*)^*$.

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- It is simple: $(C|V)^*(a|i|e)r(se)?$.
- C is an arbitrary consonant (just join all consonants with |) and V is a vowel.

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- Solution: list all that is allowed.

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- Grouping all together: $(C|V)^*((a|e|i|o|u|Vy)|V(C|V)^*(C'|C''?h))s$.

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  - Iteration (Kleene star): $L^* = \bigcup\limits_{k=0}^{\infty} L^k$.
  - $\{a, b\}^* = \{a, b\}^0 \cup \{a, b\}^1 \cup \{a, b\}^2 \cup \ldots = \{\varepsilon, a, b, aa, ab, ba, bb, \ldots\}$.

Regular expressions: what is it formally

- We distinguish regular expression α and its language $L(\alpha)$.
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Exercise: Turkish infinitives.

In Turkish there are 8 vowels:

	Front	Back
Soft	e i	a ı
Round	ü ö	u o

Infinitive is formed by suffix *-mek/-mak* attached to verb stem, where *e* appears if the last vowel of stem is front and *a* – if it is back. Write a regular expression for Turkish infinitives.

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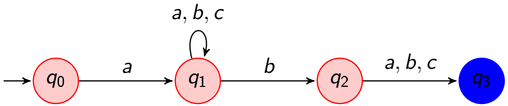
  Schematically:



- That is finite automaton.

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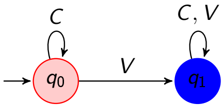
- Automaton $\mathcal{A}$ accepts language $L(\mathcal{A})$ of all words that label paths from initial state to some final.

Finite automata: examples

- A syllable: states check vowel presence.



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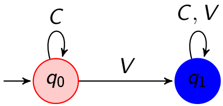

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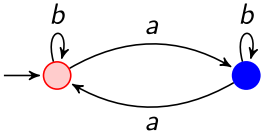

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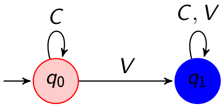

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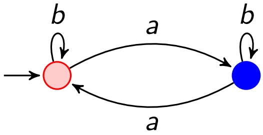

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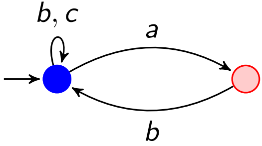

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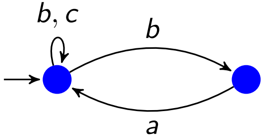

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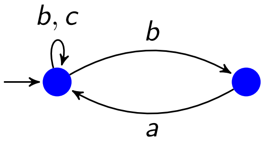
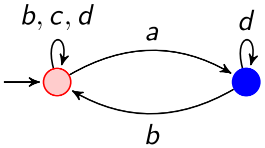


- To the right of every a occurs b with no a, c between them, alphabet a, b, c, d.



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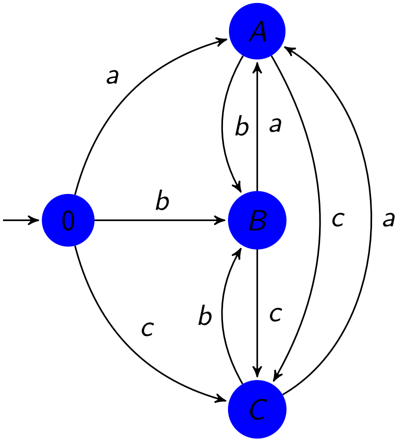
No repeating letters, alphabet a, b, c. States correspond to letters:



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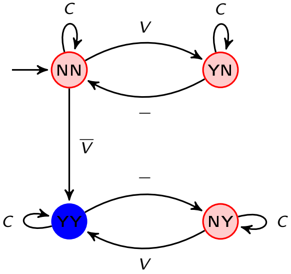

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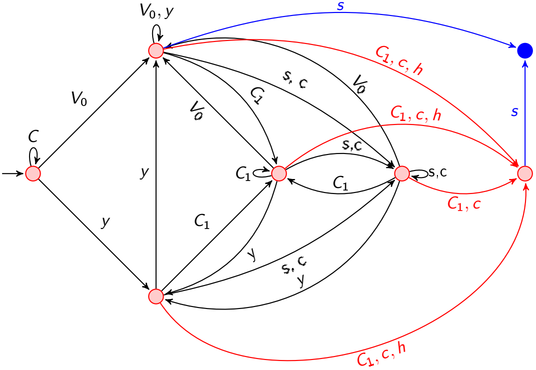
- Automaton for all possible stems
  $(C_0 = C - \{s, x, z, c, h\}, C_1 = C_0 \cup \{s, x, z\})$:



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- Add an edge $\langle q_1, a\rangle \to q_2$ if there exist states q_3, q_4 such that $(\langle q_3, a\rangle \to q_4) \in \Delta$ and there are ε-paths from q_1 to q_3 and from q_4 to q_2.

Properties of finite automata

Theorem

Every automata language is recognized by an automaton with single letter labels.

Sketch of the proof

- Split all labels of length $\geqslant 2$ by inserting additional states.
- Now we have only letters and ε as labels.
- Add an edge $\langle q_1, a\rangle \rightarrow q_2$ if there exist states q_3, q_4 such that $(\langle q_3, a\rangle \rightarrow q_4) \in \Delta$ and there are ε-paths from q_1 to q_3 and from q_4 to q_2.
- Mark as terminal all states from which terminal states are ε-reachable.
- Now remove all ε-paths.

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An automaton with one-letter labels is deterministic if no state has two outcoming edges with the same label.

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Every automata language can be recognized by deterministic automata.

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- Final states: subsets containing at least one old final state.

Kleene theorem

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The classes of automata and regular languages are the same.

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- Primitive regular languages (singletons and empty language) are certainly automata.
- We should prove that regular operations preserve automata languages.

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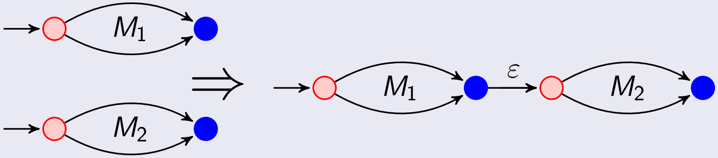

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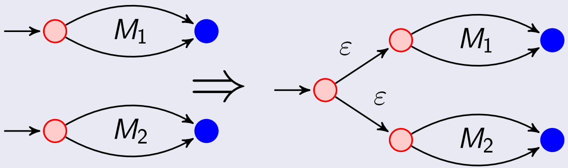
Union: $L_1 = L(M_1), L_2 = L(M_2) \to L_1 \cup L_2 = L(M)$



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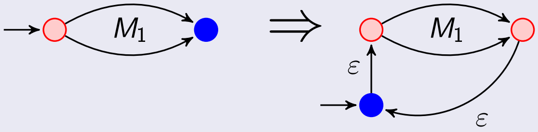
Iteration: $L_1 = L(M_1), L_1^* = L(M)$



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- Switching non-terminal and terminal states yields automaton for the complement.

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- Consider a new automaton whose states are pairs $\langle q_1, q_2 \rangle$, $q_1 \in Q_1, q_2 \in Q_2$.

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- Its start state is $\langle q_{01}, q_{02} \rangle$.
- On the first coordinate it operates like M_1, on the second like M_2.
- Finite states are pairs of final states (the automaton accepts iff it accepts for both coordinates).

Recursive construction of automata

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- For example, the automata for English plural can be expressed as:

$$(L_{sib} \cdot es) \cup (((\overline{L_{sib}} \cap L_C) \cup L_{Cy} \cup L_V) \cdot s),$$

  where

  - L_{sib} — words ending with sibilant.
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- The basic languages are the automata ones; the automaton for the whole expression could be constructed recursively.

Recursive construction of automata

Turkish infinitive

Construct a finite automaton for Turkish infinitive

- Infinitive has the form stem+*mEk*.
- Placeholder *E* is filled by *e* if the stem ends with *e*, *i*, *ö*, *ü* and *a* if it ends with *a*, *ı*, *o*, *u*.

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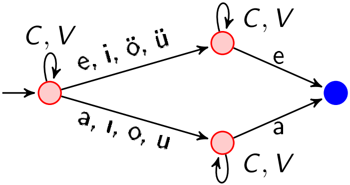
- M_1 is the automaton for expression $C^*V(C|V)^*m(a|e)k$ (it is easy to construct it).
- M_2 checks the condition for vowels:



- $M_1 \cap M_2$ is the required automaton.

Recursive construction of automata

Turkish infinitive

Construct a finite automaton for turkish passive infinitive

- Infinitive has the form stem+X+*mEk*.
- Placeholder *E* is filled by *e* if the stem ends with *e, i, ö, ü* and *a* if it ends with *a, ı, o, u*.
- Suffix *X* is -*n* if the stem ends with vowel, -*In* if the stem ends with *l* and -*Il* otherwise.
- Placeholder I equals *ı* after a, *ı*; *u* after *u, o*; *i* after *e, i*; *ü* after *ü, ö*.

Where to get presentations

- https://www.irit.fr/esslli2017/courses/33.
- http://tipl.philol.msu.ru/~otipl/index.php/department/faculty/AAS/esslli

For the next day:
Install (simply download and unpack) finite-state compiler FOMA from https://code.google.com/archive/p/foma/.