Computational morphology.
Day 2. Finite-state transducers.

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Day 2 outline

- Finite transducers.

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- Finite transducers for linguistic phenomena.

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- Compiling finite transducers with FOMA.

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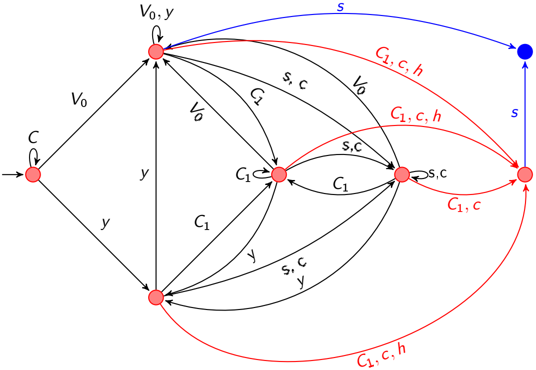
- Automaton for all possible stems
  ($C_0 = C - \{s, x, z, c, h\}$, $C_1 = C_0 \cup \{s, x, z\}$):



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- Add an edge $\langle q_1, a\rangle \to q_2$ if there exist states q_3, q_4 such that $(\langle q_3, a\rangle \to q_4) \in \Delta$ and there are ε-paths from q_1 to q_3 and from q_4 to q_2.

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- Mark as terminal all states from which terminal states are ε-reachable.
- Now remove all ε-paths.

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- Final states: subsets containing at least one old final state.

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- Primitive regular languages (singletons and empty language) are certainly automata.
- We should prove that regular operations preserve automata languages.

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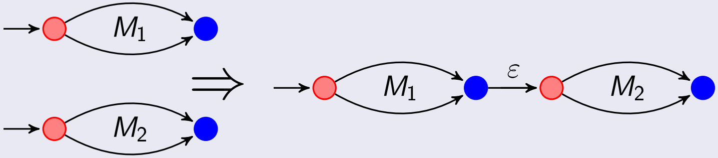

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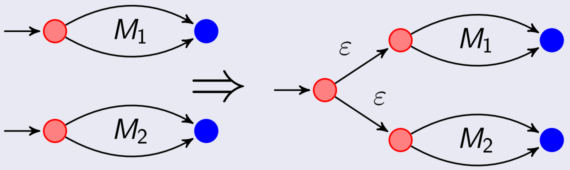
Union: $L_1 = L(M_1), L_2 = L(M_2) \rightarrow L_1 \cup L_2 = L(M)$



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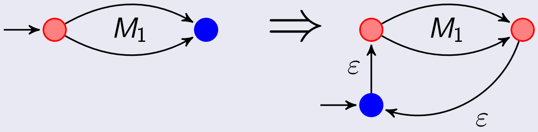
Iteration: $L_1 = L(M_1), L_1^* = L(M)$



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- Switching non-terminal and terminal states yields automaton for the complement.

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- Complex (but effective) variant: consider complete deterministic automata M_1 for L_1 and M_2 for L_2.
- Let Q_1, Q_2 be their sets of states, q_{01}, q_{02} be initial states and F_1, F_2 be sets of final states.

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- Finite states are pairs of final states (the automaton accepts iff it accepts for both coordinates).

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- For example, the automata for English plural can be expressed as:

$$(L_{sib} \cdot es) \cup (((\overline{L_{sib}} \cap L_C) \cup L_{Cy} \cup L_V) \cdot s),$$

  where

  - L_{sib} — words ending with sibilant.
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- The basic languages are the automata ones; the automaton for the whole expression could be constructed recursively.

Recursive construction of automata

Turkish infinitive

Construct a finite automaton for Turkish infinitive

- Infinitive has the form stem+*mEk*.
- Placeholder *E* is filled by *e* if the stem ends with *e*, *i*, *ö*, *ü* and *a* if it ends with *a*, *ı*, *o*, *u*.

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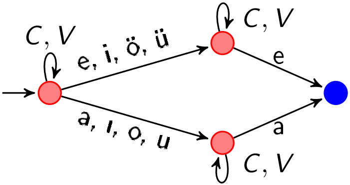
- M_2 checks the condition for vowels:



- $M_1 \cap M_2$ is the required automaton.

Recursive construction of automata

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- Suffix *X* is -*n* if the stem ends with vowel, -*An* if the stem ends with *l* and -*Al* otherwise.
- Placeholder *A* equals *ı* after a, *ı*; *u* after *u, o*; *i* after *e, i*; *ü* after *ü, ö*.

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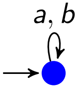

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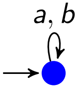
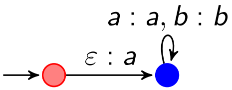


- We will formally treat finite transductions as sets of word pairs.

Finite transducers: examples

- Adds *a* to the beginning:



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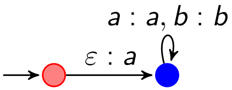

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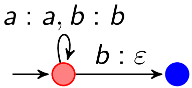

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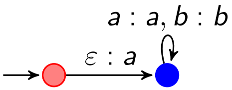

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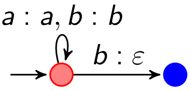
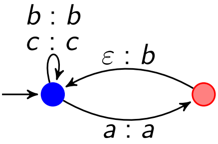


- Adds b after every a:



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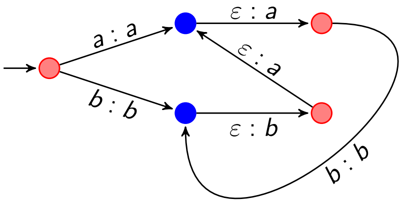

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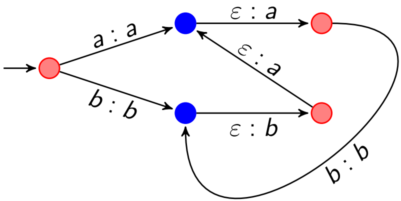
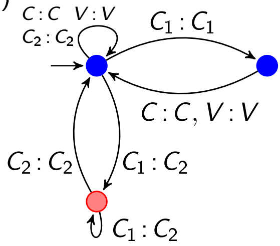


- Retro-assimilates all C_1 to C_2 (a sequence of C_1-s preceding C_2 is substituted for C_2)



Properties of finite transducers

- Every finite transducer is equivalent to a transducer with labels of the form $a : \varepsilon$, $a \in \Sigma$ and $\varepsilon : b$, $b \in \Gamma$.

Sketch of the proof

- Edges of the form $a_1 \dots a_k : b_1 \dots b_r$ can be decomposed as sequence of edges $a_1 : \varepsilon, \dots, a_k : \varepsilon, \varepsilon : b_1, \dots, \varepsilon : b_r$.

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- Finite transduction domain is an automata language (just keep only input label in the transducer).

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- Every finite transducer is equivalent to a transducer with labels of the form $a : \varepsilon$, $a \in \Sigma$ and $\varepsilon : b$, $b \in \Gamma$.

Sketch of the proof

- Edges of the form $a_1 \dots a_k : b_1 \dots b_r$ can be decomposed as sequence of edges $a_1 : \varepsilon, \dots, a_k : \varepsilon, \varepsilon : b_1, \dots, \varepsilon : b_r$.
- Edges of the form $\varepsilon : \varepsilon$ are removed as in finite automata.

- Finite transductions are closed under:
  - Concatenation.
  - Union.
  - Multiplicative iteration($\phi^* = \{u_1 \dots u_k, v_1 \dots v_k \mid \langle u_j, v_j \rangle \in \phi\}$).
- Finite transduction domain is an automata language (just keep only input label in the transducer).
- Finite transduction range is an automata language.

Properties of finite transducers

- Restriction of finite transduction to automata language can be described by finite transducer (trace both the state of the transducer and the state in the automata for the restriction language).

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Finite transducers: linguistic examples

English plural

Describe a transducer that transforms a singular form of English noun to plural.

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- *torch* $\leftrightarrows$ *torches*
- *monarch*+N+Pl $\leftrightarrows$ *monarchs*
- *ally* $\leftrightarrows$ *allies*
- *play* $\leftrightarrows$ *plays*
- *goose* $\leftrightarrows$ *geese*
- *formula* $\leftrightarrows$ *formulas/formulae*

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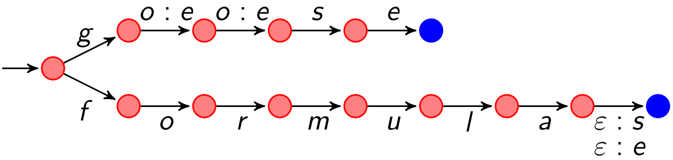
- Create a separate transducer T_{exc} for exceptions:



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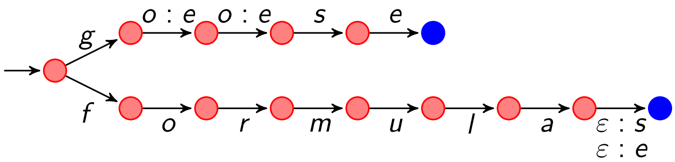
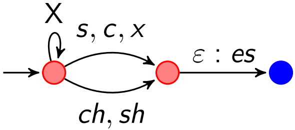


- Transducer T_{sib} that adds -*es* after word-final sibilant (X denotes any character):



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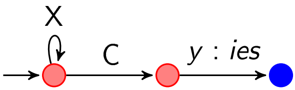

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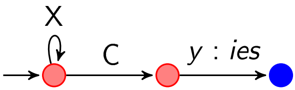

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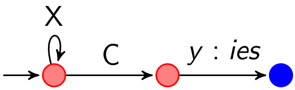

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- Final solution:

$$T_{exc} \cup_p T_{exc,sib} \cup_p T_{sib} \cup_p T_{Cy} \cup_p T_s$$

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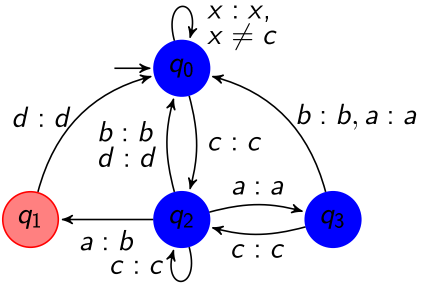

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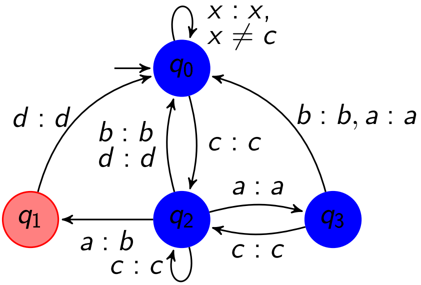

- X, Y, U, V can be arbitrary regular expressions.

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  - T_c: remove the placeholder $! \to \varepsilon$.
- Final combination via composition:

$$T_{exc} \cup_p (T_s \circ (T_{exc,sib} \cup_p T_{sib}) \circ T_y \circ T_c)$$

Turkish passive

Turkish passive

Construct a finite transducer, transforming Turkish verb infinitive to its passive infinitive.

- Passive is formed by a suffix inserted before final *-mek*/*-mak*.
- Passive suffix: *-n* after vowel, *-In* after *l* and *-Il* otherwise.
- Placeholder I: *ı* after a, *ı*; *u* after *u*, *o*; *i* after *e*, *i*; *ü* after *ü*, *ö*.
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  - *-In* after *l* (T_l): $! \rightarrow \mathrm{I}n \,||\, l\,\_\,\$$,
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  - Combine them all $T_{suf} = T_V \circ T_l \circ T_{def}$.

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- T_{mark} inserts a marker ! before *-mak/-mek*.
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- T_{fill} fills the placeholder: $T_{fill} = T_{\imath} \circ T_u \circ T_i \circ T_U$, where
  - $T_{\imath}$ checks the condition for ß: $A \to \imath \,||\, (a|\imath)C^* \_$.
  - T_u for u: $A \to u \,||\, (u|o)C^* \_$.
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  - T_U for ü: $A \to ü \,||\, (ü|ö)C^* \_$.
- Final answer:

$$T_{mark} \circ T_{suf} \circ T_{fill}$$

Nonconcatenative morphology: Yawelmani

stem	gerund	durative
caw “to cry”	caw-inay	cawaa-ʔaa-n
cuum “to destroy”	cum-inay	cumuu-ʔaa-n
hoyoo “to name”	hoy-inay	hoyoo-ʔaa-n
diiyl “to guard”	diyl-inay	diyiil-ʔaa-n
ʔilk “to sing”	ʔilk-inay	ʔiliik-ʔaa-n
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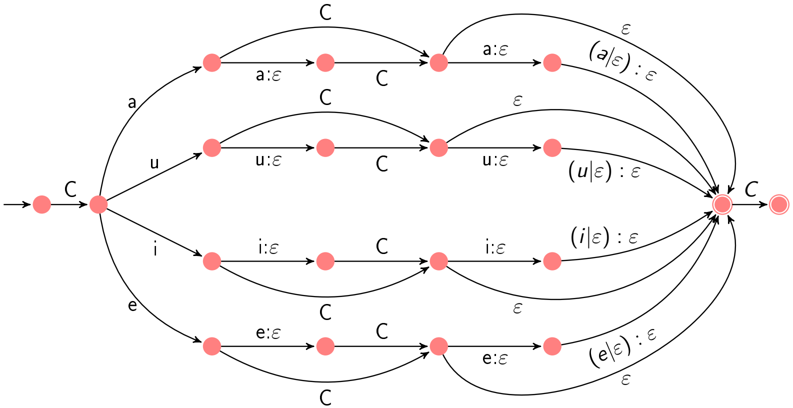
- gerund stem is $\alpha_1 V \alpha_2 \alpha_3$,
- and durative stem is $\alpha_1 V \alpha_2 VV \alpha_3$.

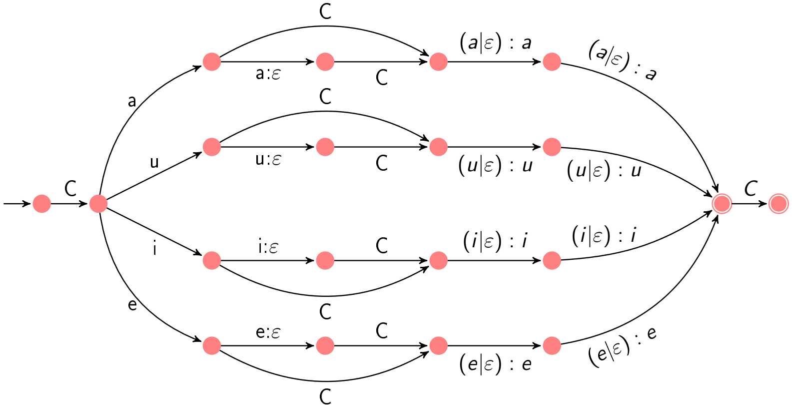
Nonconcatenative morphology: Yawelmani gerund

- Gerund stem:



Nonconcatenative morphology: Yawelmani durative

- Durative stem:



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- Flookup utility permits to use foma transducers as binary programs.

FOMA: basic usage

Basic usage: defines a context rule.

```
foma[0]: ##replace all a by b
foma[0]: regex a -> b || _ ;
374 bytes. 1 state, 3 arcs, Cyclic.
foma[1]: net
Sigma: ? @ a b
Size: 2.
Net: E20E6CF
Flags: deterministic pruned minimized epsilon_free
Arity: 2
Sfs0:   <a:b> -> fs0, b -> fs0, @ -> fs0.
foma[1]:
```

FOMA: basic usage

Basic usage: defines a context rule and applies it up and down

```
foma[0]: ##replace all a by b
foma[0]: regex a -> b || _ ;
374 bytes. 1 state, 3 arcs, Cyclic.
foma[1]: down
apply down> bcaba
bcbbb
apply down> bbb
bbb
apply down>
foma[1]: up
apply up> aba
???
apply up> cbdb
cada
cadb
cbda
cbdb
apply up> cdc
cdc
apply up>
foma[1]:
```

FOMA: basic usage

Forming plural for *y*-final nouns:

```
foma[0]: ## filter y-ending words
foma[0]: define yFinal ?* y ;
redefined yFinal: 321 bytes. 2 states, 4 arcs, Cyclic.
foma[0]: ## Vowel+y
foma[0]: define Vowel [ a | e | i | o | u ];
redefined Vowel: 413 bytes. 2 states, 5 arcs, 5 paths.
foma[0]: define yVowel [..] -> s || Vowel y _ .#. ; ## simply append s after Vowel+y
redefined yVowel: 872 bytes. 4 states, 25 arcs, Cyclic.
foma[0]: define yVowel [..] -> s || [ .#. | Vowel ] y _ .#. ; ## simply append s after Vowel+y
redefined yVowel: 872 bytes. 4 states, 25 arcs, Cyclic.
foma[0]: define yCons y -> i e s || \Vowel _ .#. ;
redefined yCons: 920 bytes. 6 states, 28 arcs, Cyclic.
foma[0]: ## combine the variants for vowels and consonants
foma[0]: define yChange yFinal .o. yVowel .o. yCons ;
redefined yChange: 936 bytes. 6 states, 29 arcs, Cyclic.
foma[0]: push yChange
936 bytes. 6 states, 29 arcs, Cyclic.
foma[1]: down
apply down> valley
valleys
apply down> ally
allies
apply down> y
ys
apply down> tray
trays
apply down> granny
grannies
```

FOMA: operations with automata

Operation	Notation
Concatenation of X, Y	XY
Intersection of X, Y	$X\&Y$
Union of X, Y	$X\|Y$
Difference of X, Y	$X - Y$
Iteration of X	X^*
Positive iteration of X	X^+
Negation of X	$\backslash X$
Context restriction	
(X appears only in context Y_Z)	$X \rightarrow Y\_Z$

Operations with automata in FOMA

FOMA: operations with automata

Operation	Notation
Context replacement	$X \rightarrow Y \| \| U\_V$
(Change X to Y in context U_V)	
Composition of X, Y	$X.\mathrm{o}.Y$
Priority union of X, Y	$X.\mathrm{P}.Y$
Cartesian product of X, Y	$X : Y$
Domain (upper part) of X	$X.u$
Range (lower part) of X	$X.l$
Inverse transduction of X	$X.i$
Parallel contexts	$X \rightarrow Y \| \| U1\_V1, U2\_V2$
Parallel replacement	$X1 \rightarrow Y1, X2 \rightarrow Y2 \| \| U\_V$

Operations with transducers in FOMA

FOMA: applying transducers

Operation	Notation
Define a transducer variable	**define** ⟨var_name⟩ ⟨expression⟩
Push defined transducer to the stack	**push** ⟨var_name⟩
Push expression to the stack	**regex** ⟨expression⟩
Apply topmost transducer in stack (downwards)	**down** (apply down)
Apply topmost transducer "reversely" (upwards)	**up** (apply up)
Clear stack	**clear**
Read lexicon file and save to variable	**read** ⟨filename⟩ **define** ⟨var_name⟩
save transducer(s) to binary file	**save stack** ⟨filename⟩

Application of transducers in FOMA

FOMA: external usage and documentation

- Documentation page (concise but useful): `https://code.google.com/archive/p/foma/wikis`.
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- Main usage:

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- Applies the transducer in binary file to each string in ⟨input_file⟩ and prints the result (or redirects it to ⟨output_file⟩).

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- More documentation: `https://code.google.com/archive/p/foma/wikis/FlookupDocumentation.wiki`.

English plural

```
### english.foma ###
read lexc irregular.lexc
define IrregularNounPlural;

define Vowel [ a | i | e | o | u | y ];
define Consonant [ b | c | d | f | g | h | j | k | l | m | n | p | q | r | s | t | v | w | x | z ];
define Letter [Vowel | Consonant];
define Word [ Letter ]+;
define NounMark "+N";
define NounNumber "+Sg" | "+Pl";
define Noun Word NounMark NounNumber;

define NounAffixation "+N" "+Sg" -> "" || _ .#., "+N" "+Pl" -> "!" s || _ .#.;
define Sibilant [ x | s | z | c h | s h ];
define sibException [ Letter ]+ a r c h "!" s ;
define eInsertion [..] -> e || Sibilant _ "!" s .#.;
define checkSibilant [ sibException .P. eInsertion ];
define yReplacement y -> i e || Consonant _ "!" s .#.;
define Cleanup "!" -> "" || _ ;
define RegularNoun [ NounAffixation .o. yReplacement .o. checkSibilant .o. Cleanup ] ;
define Grammar Noun .o. [ IrregularNounPlural .P. RegularNoun ];
push Grammar
```

Turkish passive

Turkish passive

Construct a finite transducer, transforming Turkish verb infinitive to its passive infinitive.

- Passive is formed by a suffix inserted before final *-mek/-mak*.
- Passive suffix: *-n* after vowel, *-In* after *l* and *-Il* otherwise.
- Placeholder I: *ı* after a, *ı*; *u* after *u, o*; *i* after *e, i*; *ü* after *ü, ö*.

```
# symbol classes
define HardStraightVowel a | I ;
define HardRoundVowel o | u ;
define SoftStraightVowel e | i ;
define SoftRoundVowel O | U ;
define HardVowel HardStraightVowel | HardRoundVowel ;
define SoftVowel SoftStraightVowel | SoftRoundVowel ;
define Vowel HardVowel | SoftVowel ;
define Consonant b | c | C | d | f | g | G | h | j | k | l | m | n | p | r | s | S | t | v | y | z ;
define Letter Consonant | Vowel ;
```

Turkish passive

```
# contexts for stem
define LastVowelHard HardVowel Consonant* ;
define LastVowelSoft SoftVowel Consonant* ;
define LastVowelHardRound HardRoundVowel Consonant* ;
define LastVowelHardStraight HardStraightVowel Consonant* ;
define LastVowelSoftRound SoftRoundVowel Consonant* ;
define LastVowelSoftStraight SoftStraightVowel Consonant* ;
# infinitive vowel check
define Stem Letter* Vowel Letter* ;
define InfinitiveSuffixInsertion [..] -> E || _ .#. ;
define InfinitiveSuffix [ E -> m a k || HardVowel Consonant* _ .#. ] .o. [ E -> m e k || SoftVowel
    Consonant* _ .#. ] ;
define SuffixTransform Stem .o. InfinitiveSuffixInsertion .o. InfinitiveSuffix ;
define Infinitive SuffixTransform.l ;
define Input Infinitive "+Pass";

# suffix insertion
define MarkerInsertion [..] -> "!" || _ m [ a | e ] k "+Pass" .#. ;
define MarkerAfterVowel "!" -> l || Vowel _ ;
define MarkerAfterL "!" -> A n || l _ ;
define MarkerAfterAll "!" -> A l || _ ;
define MarkerReplacement MarkerAfterVowel .o. MarkerAfterL .o. MarkerAfterAll ;

# combining all
define VowelFill [ A -> I || LastVowelHardStraight _ ] .o. [ A -> e || LastVowelSoftStraight _ ] .o. [
    A -> u || LastVowelHardRound _ ].o. [ A -> U || LastVowelSoftRound _ ] ;
define Cleanup "+Pass" -> "" ;
define Grammar Input .o. MarkerInsertion .o. MarkerReplacement .o. VowelFill ;

push Grammar
```

Yawelmani verb forms

stem	gerund	durative
caw "to cry"	caw-inay	cawaa-ʔaa-n
cuum "to destroy"	cum-inay	cumuu-ʔaa-n
hoyoo "to name"	hoy-inay	hoyoo-ʔaa-n
diiyl "to guard"	diyl-inay	diyiil-ʔaa-n
ʔilk "to sing"	ʔilk-inay	ʔiliik-ʔaa-n
hiwiit "to walk"	hiwt-inay	hiwiit-ʔaa-n

Verb forms in Yawelmani (Amerind family)

If the stem was $\alpha_1 V(V)\alpha_2(V)(V)\alpha_3$, where $\alpha_1, \alpha_2 \in C$, $\alpha_3 \in \{C, \varepsilon\}$:

- gerund stem is $\alpha_1 V \alpha_2 \alpha_3$,
- and durative stem is $\alpha_1 V \alpha_2 V V \alpha_3$.

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  - Check vowel harmony between syllables by enumerating all variants of the type $C^+xC^+xC^*$ where x is an arbitrary vowel.

Yawelmani verb forms

```
### youlumne.foma ###
define Vowel [a | i | o | u];
define Consonant [ c | w | m | h | y | d | l | g | k | t ];
define Letter [Consonant | Vowel];
define Stem Consonant Vowel (Vowel) Consonant (Vowel) (Vowel) (Consonant) ;
define VerbMark "+V";
define Mood "+Ger" "+Dur";
define Mark [ VerbMark Mood ];
# vowel harmony
define VowelPattern [ [Consonant | a]+ | [Consonant | i]+ | [Consonant | o]+ | [Consonant | u]+ ];
define Word Stem & VowelPattern;
# left context for not a leftmost vowel
define LeftContext1 Consonant Vowel [ Letter ]* ;
define VowelRemoval Vowel -> [] || LeftContext1 _ ;
# left context for second syllable vowels
define LeftContext2 Consonant Vowel Consonant+ ;
# durative vowel insertion
define DurativeVowelInsertion [..] -> [ a a | i i | o o | u u ] || LeftContext2 _ (Consonant) .#. ;
define GerundSuffixInsertion ["+V" "+Ger"] : [ i n a y ] ;
define DurativeSuffixInsertion ["+V" "+Dur"] : [ g a a n ] ;
define GerundStem Word .o. VowelRemoval ;
# check that word possesses vowel harmony after vowel insertion
define DurativeStem GerundStem .o. DurativeVowelInsertion .o. VowelPattern ;
define Gerund [ GerundStem GerundSuffixInsertion ] ;
define Durative [ DurativeStem DurativeSuffixInsertion ] ;
define Grammar [ Gerund | Durative ] ;
```